1. Details of Module and its structure

Module Detail	
Subject Name	Physics
Course Name	Physics 01 (Physics Part-1, Class XI)
Module Name/Title	Unit 4, Module 4, Spring and Force Constant Chapter 6,Work,Energy and Power
Module Id	Keph_10604_eContent
Pre-requisites	Kinematics, laws of motion, basic vector algebra potential energy, kinetic energy, work energy theorem
Objectives	After going through this module, the learners will be able to: • Understand the special features of a spring • Plot a Load extension graph • Deduce the meaning of Force constant • Calculate Potential energy stored in a compressed or stretched spring • Recognize the use of springs in real life
Keywords	Potential energy Spring, spring constant, load extension graph, potential energy stored in a spring

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1. UNIT SYLLABUS

UNIT IV: Chapter 6: WORK ENERGY AND POWER

Work done by a constant force and a variable force; kinetic energy; work energy theorem; power; Notion of potential energy; potential energy of a spring conservative and non-conservative forces; conservation of mechanical energy (kinetic and potential energies) non-conservative forces; motion in a vertical circle; Elastic and inelastic collisions in one and two dimensions.

2. MODULE-WISE DISTRIBUTION OF UNIT SYLLABUS 7 Modules

This unit is divided into 7 modules for better understanding.

Module 1	• Meaning of work in the physical sense • Constant force over variable displacement • variable force for constant displacement • Calculating work • Unit of work • Dot product • Numerical
Module 2	• Kinetic energy • Work energy theorem • Power • Numerical
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Module 4	• Potential energy • Elastic Potential energy • Springs • Spring constant • Problems
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MODULE 4

3. WORDS YOU MUST KNOW

Let us keep the following concepts in mind

- Rigid body: An object for which individual particles continue to be at the same separation over a period of time.

- Point object: **Point object** is an expression used in kinematics: it is an **object** whose dimensions are ignored or neglected while considering its motion.
- Distance travelled: change in position of an object is measured as the distance the object moves from its starting position to its final position. Its SI unit is m and it can be zero or positive.
- Displacement: a **displacement** is a vector whose length is the shortest distance from the initial to the final position of an object undergoing motion. . Its SI unit is m and it can be zero, positive or negative.
- Speed: Rate of change of position .Its SI unit is ms^{-1}.
- Average speed: $\frac{\text{total path length travelled by the object}}{\text{total time interval for the motion}}$

  Its SI unit is ms^{-1}.
- Velocity (v): Rate of change of position in a particular direction.
  Its SI unit is ms^{-1}.
- Instantaneous velocity: velocity at any instant of time.

$$v_{instaneous} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Instantaneous velocity is the **velocity** of an object in motion at a specific time. This is determined by considering the time interval for displacement as small as possible .the instantaneous velocity itself may be any value .If an object has a constant **velocity** over a period of time, its average and **instantaneous velocities** will be the same.

- Uniform motion: a body is said to be in uniform motion if it covers equal distance in equal intervals of time
- Non uniform motion: a body is said to be in non- uniform motion if it covers unequal distance in equal intervals of time or if it covers equal distances in unequal intervals of time

- Acceleration (a): time rate of change of velocity and its SI unit is ms^{-2}. Velocity may change due to change in its magnitude or change in its direction or change in both magnitude and direction.
- Constant acceleration: Acceleration which remains constant throughout a considered motion of an object
- Momentum (p): The impact capacity of a moving body. It depends on both mass of the body and its velocity. Given as p = mv and its unit is kg ms^{-1}.
- Force (F): Something that changes the state of rest or uniform motion of a body. SI Unit of force is Newton (N). It is a vector, because it has both magnitude, which tells us the strength or magnitude of the force and direction. Force can change the shape of the body.
- Constant force: A force for which both magnitude and direction remain the same with passage of time
- Variable force: A force for which either magnitude or direction or both change with passage of time
- External unbalanced force: A single force or a resultant of many forces that act externally on an object.
- Dimensional formula: An expression which shows how and in which way the fundamental quantities like, mass (M), length (L) and time (T) are connected
- Kinematics: Study of motion of objects without involving the cause of motion.
- Dynamics: Study of motion of objects along with the cause of motion.
- Vector: A physical quantity that has both magnitude and direction .displacement, force, acceleration are examples of vectors.
- Vector algebra: Mathematical rules of adding, subtracting and multiplying vectors.
- Resolution of vectors: The process of splitting a vector into various parts or components. These parts of a vector may act in different directions. A vector can be resolved in three mutually perpendicular directions. Together they produce the same effect as the original vector.
- Dot product: If the product of two vectors (A and B) is a scalar quantity. Dot product of vector A and B: A.B= $|A||B|cos\theta$ where θ is the angle between the two vectors

Since Dot product is a scalar quantity it has no direction. It can also be taken as the product of magnitude of A and the component of B along A or product of B and component of A along B.

- Work: Work is said to be done by an external force acting on a body if it produces displacement W= F.S cosθ, where work is the dot product of vector F(force) and Vector S (displacement) and θ is the angle between them . Its unit is joule and dimensional formula is ML^2T^{-2}. It can also be stated as the product of component of the force in the direction of displacement and the magnitude of displacement. Work can be done by constant or variable force and work can be zero, positive or negative.

- Energy: The ability of a body to do work

- Kinetic Energy: The energy possessed by a body due to its motion = ½ mv^2, where 'm' is the mass of the body and 'v' is the velocity of the body at the instant its kinetic energy is being calculated.

- Work Energy theorem: Relates work done on a body to the change in mechanical energy of a body i.e.,

  $W = F.S = \frac{1}{2} mV_f^2 - \frac{1}{2}mV_i^2$

- Conservative force: A force is said to be conservative if the work done by the force in displacing a body from one point to another is independent of the path followed by the particle and depends on the end points. Example: gravitational force.
- Non- conservative forces: If the amount of work done in moving an object against a force from one point to another depends on the path along which the body moves, then such a force is called a non-conservative force. Example: friction.
- Conservation of mechanical energy: Mechanical energy is conserved if work done is by conservative forces.

- Potential energy due to position: Work done in raising the object of mass m to a particular height (distance less than radius of the earth) = ‘mgh’.

4. INTRODUCTION

When we try to stretch or compress an elastic object like a rubber band, catapult, spring etc, it gets deformed but once the external deforming force is removed it gets back to its original shape. This is due to the change in the inter atomic/molecular separation.
The intermolecular forces give rise to a restoring force, which tries to restore the object to its original shape and size. **This restoring force is equal and opposite to the external force applied on the object and is found to be conservative in nature.**

The work done, against this restoring force, gets stored in the object in the form of its elastic potential energy.

The more we stretch or compress an object, more is the elastic potential energy stored in the object. But if we pull or compress too hard we may deform it complexly and permanently. So within limits the springiness, also called elastic limit, it is possible to deform a body and restore it back to its original shape and size.

Do you think shape could also be responsible?

Consider a wire, it does not behave like the spring made out of the same wire.

Let us study springs in a little more detail.

5. SPRINGS

We have seen springs in our pens, toys etc.

Spring is a wire of uniform area of cross section coiled closely and with a constant cylindrical shape

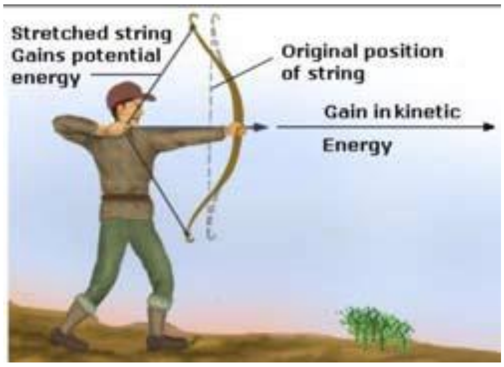
We have experienced that we can compress or stretch a spring.

The fact that they come back “spring back” to their original shape, once the external forces are removed, gives them a large number of applications in real life.

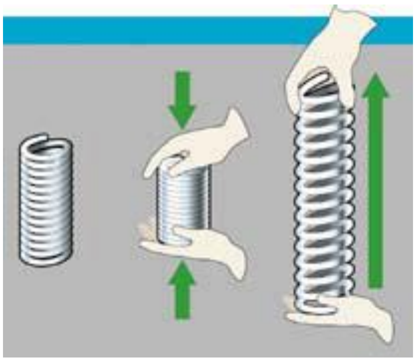
The energy which gets stored up by deformation of a spring is called elastic potential energy.

SOME EXAMPLES OF ELASTIC POTENTIAL ENERGY ARE AS FOLLOWS:

a) **When a bow is drawn, the work done gets stored in the form of its elastic potential energy**



b) **The compressed springs, of a dart gun, store elastic potential energy. On pressing the trigger, this potential energy of springs changes to kinetic energy of the dart.**



c) **The work done, in pulling the rubber band of a sling shot, cut out of cycle tube, gets stored in it as its elastic potential energy. This can manifest itself in the form of the kinetic energy of the missile.**

d) **In a pole vault, the kinetic energy of the player changes into elastic potential energy of the pole.**

https://www.youtube.com/watch?v=ozAeIO46lAw

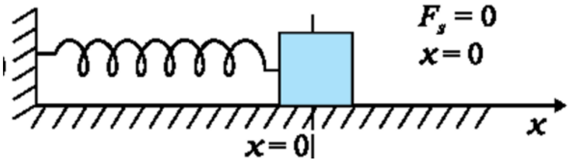
Beijing Olympics 2015

e) **Springs, present in the shock absorbers of vehicles, convert the kinetic energy of the vehicle into their elastic potential energy. This enables us to have a safe ride over the speed breakers on the roads.**

6. ELASTIC POTENTIAL ENERGY IN A SPRING

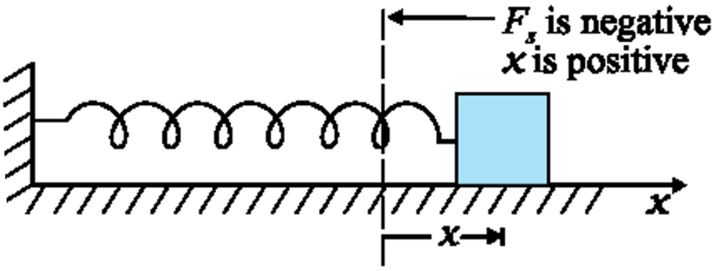
Springs store elastic potential energy when stretched or compressed.

Consider a block of mass *m* which is lying on a frictionless horizontal surface and is attached to one end of a mass less (negligible mass) elastic horizontal spring.



The other end of this spring is fixed to a rigid support. The position of the block, when it is at rest and the spring is un-stretched, is taken as the origin (x = 0) and **it is the mean or equilibrium position.**

When the block is pulled towards right, by applying a horizontal force F in the positive direction of the x-axis, the spring gets stretched.



Due to elasticity, a restoring force gets developed in the spring which tries to bring the spring back to its un-stretched or equilibrium position.

According to Newton's third law of motion this restoring force is equal and opposite to the applied force and is, therefore, directed along the negative x-axis.

It is directly proportional to the displacement of the block from the mean position.

We say so because, for elastic springs, the force needed to stretch them, through a distance x, is directly proportional to x itself.

Hence, F $\propto x$

Or F = – kx

Where k is the constant of proportionality, is known as the spring constant of the spring.

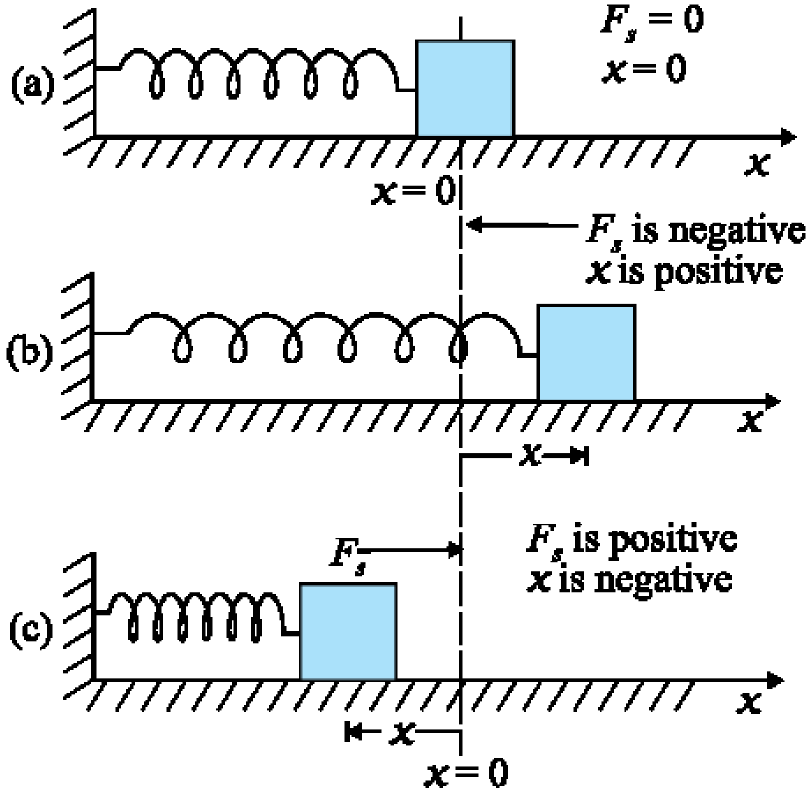


Fig shows a block attached to a spring and resting on a smooth horizontal surface. The other end of the spring is attached to a rigid wall. The spring is light and may be treated as negligible mass.

What makes the spring compress?

How much displacement takes place in the negative x direction ?

7. MEASURING SPRING CONSTANT IN THE LABORATORY

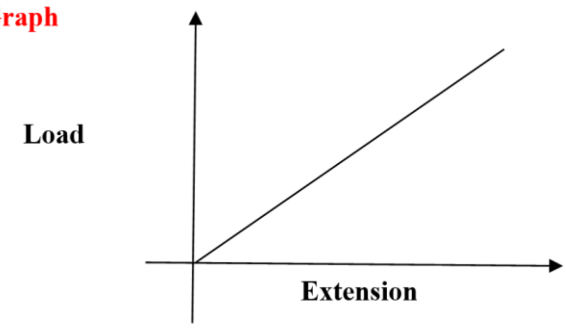
Let us do a simple experiment:

Load extension graph can be plotted using an apparatus with a vertical spring which extends on loading

Loading the spring in steps, by using slotted weights of 50 g each

Load (g)	Extension(cm)
0	1
50	2
100	3
150	4
200	5
250	6

Graph



The **slope of this straight line** graph is **force/extension = k**

Wht would happen to K

- **if the external force is reduced or removed?**
- **If the spring goes back to its original shape and size.**

8. FORCE CONSTANT

THINK ABOUT THESE

- **The relation F = – *k x* is valid only for small displacements.**
- **The negative sign in this expression is there because the restoring force is always opposite to the direction of the displacement**
- $|\boldsymbol{k}| = \frac{|\boldsymbol{F}|}{|\boldsymbol{x}|}$
- **The value can be obtained from load extension graph.**
- **Spring constant is numerically equal to the restoring force per unit displacement**
- **S.I. unit is Nm^{-1}**
- ***k* is also known as stiffness constant of a spring., a spring is stiffer if k is large .**
- **The spring constant of a spring depends on its material , way a spring is made, diameter of spring diameter of wire used for the spring**

We see that

The restoring **force F = - k x is not a constant force. It is maximum, when the spring is deformed the most, and is zero when it is not deformed**.

Thus, **the spring force or restoring force in a spring is an example of a variable force which is conservative**.

9. ELASTIC POTENTIAL ENERGY

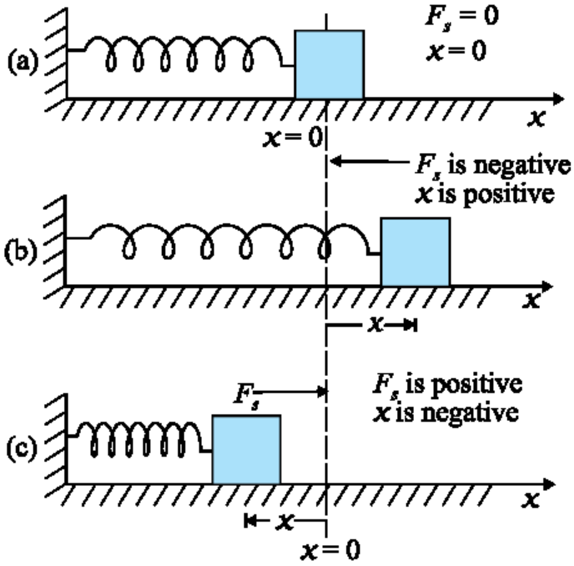


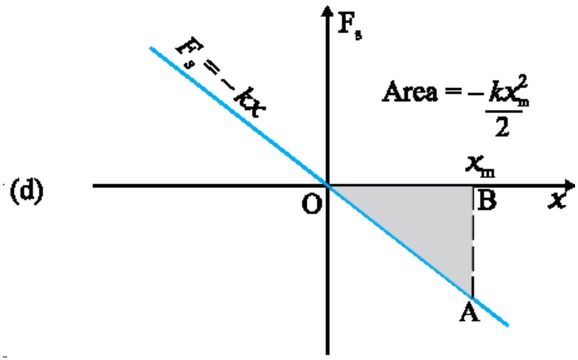
Illustration of the spring force with a block attached to the free end of the spring.

(a) The spring force F_s is zero when the displacement x from the equilibrium position is zero.

(b) For the stretched spring x > 0 and Fs < 0

(c) For the compressed spring x < 0 and $F_s > 0$.

(d) The plot of F_s versus x.



The area of the shaded triangle represents the work done by the spring force. Due to the opposing signs of F_s and x, this work done is negative,

$$W_s = -\frac{kx_m{}^2}{2}$$

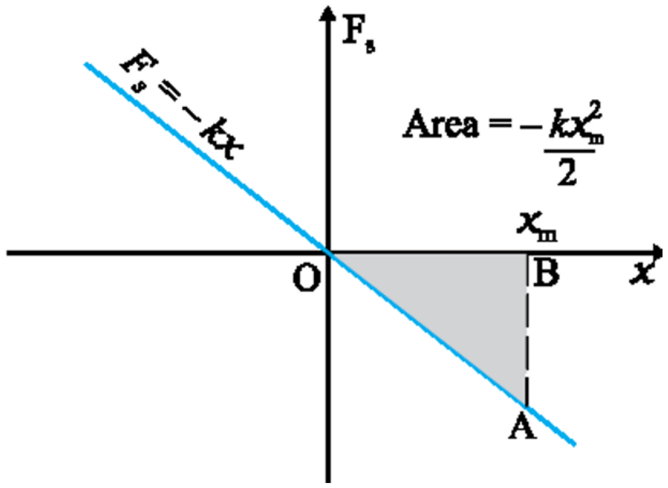
So we can integrate

$$W_s = \int_0^{x_m} F_s\,dx = -\int_0^{x_m} kx dx$$

$$= -\frac{\mathbf{kx_m{}^2}}{\mathbf{2}}$$

We can obtain the same value by using average force and then calculating the work done by this average force for a displacement of x .

From the graph



Initial force = 0

maximum force = - kx or magnitude of force = |kx|

Average = $\frac{0+kx}{2}$

Work = $\frac{0+kx}{2} \times x$

$= \frac{1}{2}kx^2$

Which the same is as obtained earlier.

This is the work done by the spring force. Hence the work done by the external agent, which will get stored in the spring as its potential energy

We can say the object can possess two types of mechanical energies:

1. **Kinetic energy due to its motion.**
2. **Potential energy due to:**

a) Position above a horizontal level.

b) Configuration which is temporary change in its shape.

10. ANOTHER METHOD TO FIND THE FORCE CONSTANT OF A SPRING

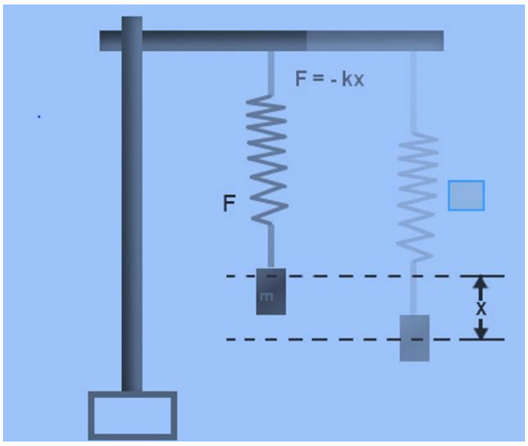
To find the force constant and unknown mass of a body by using the oscillations of a helical spring by plotting T^2 – m graph.

You can derive a relation between periodic time (T), spring constant (k) and mass (m) using the method of dimensions

This relation is

$$\mathbf{T = 2\pi\sqrt{\frac{m}{k}}}$$

Experimentally we find time for 20 oscillations and calculate the periodic time.



Note

the mass on the hanger and calculate spring constant.

You can now use the additional object whose mass m` is to be determined along with the initially loaded mass (m) and find the periodic time T` of the assembly. Later make calculations for just the unknown mass

$$T' = 2\pi\sqrt{\frac{m+m'}{k}}$$

11. SOME EXAMPLES

EXAMPLE:

To simulate car accidents, auto manufacturers study the impact of moving cars with mounted springs of different spring constants. Consider a typical simulation of a car of

mass 1000kg. **So imagine small cars like Alto, Ritz, Santro, Polo etc.** **Moving with a speed of 18 km/h on a smooth road and colliding with a horizontally mounted spring of spring constant $\boldsymbol{k = 6.25 \times 10^3 Nm^{-1}}$. what is the maximum compression of the spring?**

SOLUTION:

At maximum compression the kinetic energy of the car is converted entirely into the potential energy of the spring.

The kinetic energy of the moving car is

We must remember that 36 km/h = 10m/s

So our car is moving at 5 m/s

KE $= \frac{1}{2}mv^2 = \frac{1}{2}1000 \times 5 \times 5$

$$\boldsymbol{= 1.25 \times 10^4 \mathrm{J}}$$

Potential energy in a spring $= \frac{1}{2}kx^2 = \frac{1}{2}6.26 \times 10^3 x^2$

Solving the two equations

x = 2.0 m

This is the basis for making airbags for safety

WE ARE ASSUMING:

- **No other form of energy is generated so no sound, no heat**
- **The spring is considered mass less**
- **The road is considered friction less**
- **No time has been fixed for the compression of the spring**

EXAMPLE:

A spring has a force constant of 24 N m^{-1}. A mass of 4 kg is attached to the spring. The two are at rest on a horizontal frictionless table. If a constant force of 10 N is applied on the block. What is the speed of the block when it is at a displacement of 0.5 m?

SOLUTION:

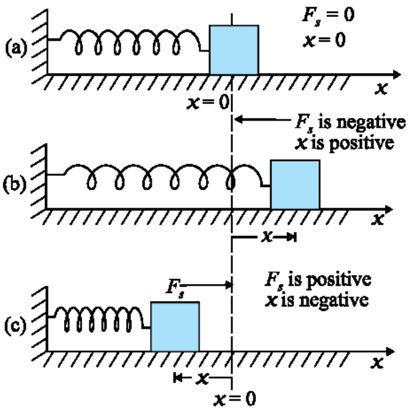
Using the above diagram as reference

k = 24 Nm^{-1}

m = 4 kg,

x = 0.5 m

Work done = sum of mechanical energy



$$10 \times 0.5 = \frac{1}{2} \times 4 \times v^2 + \frac{1}{2} \times 24 \times 0.25$$

$$5 = 2v^2 + 3$$

Or **v = 1 ms^{-1}**

12. COMMON MISCONCEPTIONS

Work done by the spring force is always negative.

But it is not so.

The work done by the spring force may be zero, positive or negative depending on the situation.

Case 1: When the block reaches its mean position, the spring is in its un-stretched position. Then $x = 0$ and spring force is zero and thus work = 0.

Case 2: If the applied force is stretching, or compressing, the spring then spring force, being restoring in nature, acts towards the mean position. So the work done by the spring force is negative.

Case 3: When the block is released after the spring is stretched or compressed then the block starts moving towards the mean position and the work done by the spring force is positive.

13. SUMMARY

We sum up our study of springs as:

- The design of the spring from a wire of uniform diameter ,makes it possible to store energy on compression or elongation.

- The spring factor is important to calculate the ability of a spring to store energy.
- Springs are used in daily life because they can store mechanical energy.